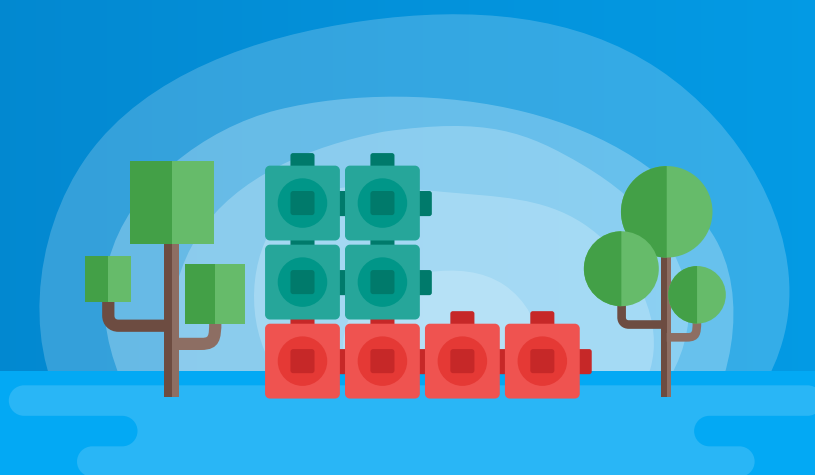


IMPROVING MATHEMATICS IN THE EARLY YEARS AND KEY STAGE 1

Guidance Report



Education
Endowment
Foundation

We would like to thank the many researchers and practitioners who provided support and feedback on drafts of this guidance. In particular, we would like to thank the advisory panel and evidence review team:

Advisory panel: Dr Sue Gifford (University of Roehampton), Dr Terezinha Nunes (University of Oxford), Mari Palmer (North Yorkshire Coast Research School), and Viv Lloyd (NCETM).

Evidence review team: Professor Jeremy Hodgen (Institute of Education, University College London), Dr Nancy Barclay (The University of Brighton), Dr Colin Foster (Loughborough University), Dr Camilla Gilmore (Loughborough University), Dr Rachel Marks (University of Brighton), and Dr Victoria Sims (Ulster University).

Guidance report authors: Amy Clark (EEF) and Peter Henderson (EEF). Dr Sue Gifford (University of Roehampton) authored most of the examples.

About the Education Endowment Foundation

The Education Endowment Foundation (EEF) is an independent charity supporting teachers and school leaders to use evidence of what works—and what doesn't—to improve educational outcomes, especially for disadvantaged children and young people.

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FOREWORD



Mathematics plays a key role in a child's development. Very young children are naturally curious, noticing differences in quantity and the shape of objects, and use early mathematical concepts when they play. Mathematical understanding helps children make sense of the world around them, interpret situations, and solve problems in everyday life, whether that's understanding time, sharing amounts with their peers, or counting in play.

Developing a sound understanding of mathematics when we are young is essential.

Children's early mathematical understanding is strongly associated with their later school achievement.¹ It has, therefore, a major impact on young people's educational progress and life outcomes.

“Developing a sound understanding of mathematics when we are young is essential.”

Yet not all children learn the skills they need to succeed. In 2018, just 66% of disadvantaged children achieved at least the expected level of development for number at the end of the Early Years Foundation Stage compared to 82% of their peers. Once children fall behind, it is

hard for them to catch up and they are likely to fall further behind throughout school.

It is crucial, then, that we start early and make sure that all young people—regardless of background—have access to great mathematics teaching in the early years and at primary school.

This is why the EEF have produced this guidance report. It offers five practical recommendations to support the learning of children aged 3 to 7 in the early years and Key Stage 1. To develop these recommendations, we reviewed the best available international research and consulted experts to arrive at key principles for effective practice. These are illustrated with examples to help practitioners put the evidence to good use.

This is a companion to our other guidance report, [‘Improving Mathematics in Key Stages 2 and 3’](#), supporting the learning of 7–14 year-olds. As with all EEF guidance reports, publication is just the start. We will now be working with the early years and primary sector, including through our colleagues in the Research Schools Network, to build on the recommendations with further training and resources.

Our hope is that this guidance will support consistently excellent, evidence-informed early years and primary provision that creates great opportunities for all children, whatever their background.



Prof Becky Francis

Chief Executive
Education Endowment Foundation

What does this guidance cover?

This report is part of a series that the EEF is producing on mathematics. It focuses on children aged three to seven in the early years and Key Stage 1. A previous EEF report covers key recommendations for teaching mathematics in Key Stages 2 and 3.²

There are several advantages to addressing both the early years and Key Stage 1 in the same report:

- The two phases share common ‘best bets’ for effective mathematics teaching. While the two phases might have different pedagogical styles and curricula, and so these recommendations require careful application across different contexts, the key recommendations for improving mathematics teaching apply across both phases.
- A deeper understanding of practice in each phase could support practitioners’ collaboration and improve the transition between phases.
- Training programmes based on this guidance can involve practitioners from both early years and Key Stage 1. Bringing the two phases together will allow them to learn from each other’s strengths.

The term ‘practitioner’ is used throughout this guidance to refer to all educators across the early years and Key Stage 1. However, in some examples, where appropriate to the context, other terms are used.

This guidance draws largely on a review of the evidence on mathematics teaching for ages three to seven conducted by Professor Jeremy Hodgen, Dr

Nancy Barclay, Dr Colin Foster, Dr Camilla Gilmore, Dr Rachel Marks, and Dr Victoria Sims. More information about how this guidance was created is available at the end of the report.

This report is not intended to provide a comprehensive guide to mathematics teaching and learning. Rather, the recommendations focus on particular areas where there is evidence that certain practices can make a significant difference to children’s learning. The report does not cover all of the potential components of successful mathematics provision. Some aspects will be missing because they are related to organisational or leadership issues, or the content of curricula; other areas are not covered because there is insufficient robust evidence to create an actionable recommendation.

Unfortunately, there is currently little robust evidence concerning the effectiveness of particular approaches to supporting mathematics learning during transitions, for example from Reception to Year 1. Furthermore, despite parental engagement in early years education being consistently associated with children’s subsequent academic success,³ less is currently known about how to intervene to support children’s numeracy through working with parents. The EEF’s [‘Working with Parents to Support Children’s Learning’](#) guidance report summarises the best available research on supporting settings to work with parents.⁴

“Bringing the two phases together will allow them to learn from each other’s strengths”

INTRODUCTION

Who is this guidance for?

This guidance is aimed primarily at mathematics leaders, senior leaders, and other staff with leadership responsibility for the early years or Key Stage 1. This includes a variety of early years settings. Some elements of the guidance may be relevant to younger children in early years settings, however the evidence that fed into this guidance was for children aged three to seven, so this age-range is the focus here. Senior leaders have responsibility for managing change across a setting so attempts to implement these recommendations are more likely to be successful

if they are involved. Practitioners will also find this guidance useful as a resource to aid their day-to-day practice.

It may also be used by:

- governors and parents to support and challenge staff; and
- those that work with settings, schools, and practitioners to provide training and support.

What support is available for using this guidance?

The Research Schools Network is a collaboration between the EEF and the Institute for Effective Education to fund a network of schools and settings that support the use of evidence to improve teaching practice. The network contains early years settings, primary schools, and secondary schools.

Research Schools work with the other schools and settings in their areas to help them make better use of evidence to inform their teaching by:

- encouraging schools and settings to make use of **evidence-based programmes and practices** through regular communication and events;
- providing **training and professional development** for senior leaders and practitioners on how to improve practice based on the best available evidence; and
- supporting schools and settings to develop innovative ways of improving teaching and learning and provide them with the **expertise to evaluate their impact**.

More information about the Research Schools Network, and how it can provide support on the use of EEF guidance reports, can be found at <https://researchschool.org.uk/>. In addition, the EEF has six regional teams across the country that help foster and coordinate school improvement partnerships with Local Authorities, Multi-Academy Trusts, Teaching School Alliances and informal groups of schools and settings.

The EEF will also produce a number of additional resources that will sit alongside this guidance report to support practitioners to build on these recommendations and put them into practice.

If you have examples of a recommendation that has been effectively implemented in your setting, then please get in touch: info@eefoundation.org.uk.

Acting on the guidance

There are several key principles to consider when acting on this guidance.

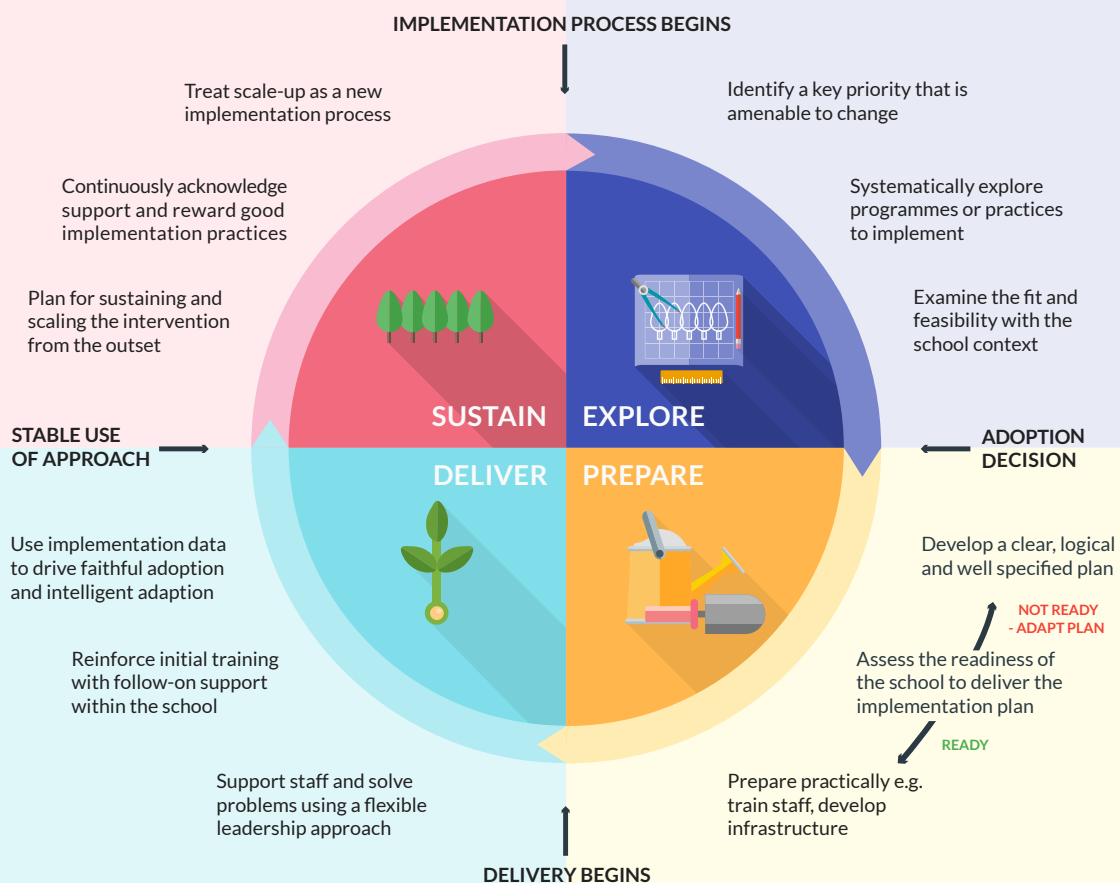
These recommendations do not provide a ‘one size fits all’ solution. It is important to consider the delicate balance between implementing the recommendations faithfully and applying them appropriately to your setting’s particular context. Implementing the recommendations effectively will require careful consideration of how they fit your setting’s context and the application of sound professional judgement.

The recommendations should be considered together, as a group, and should not be implemented selectively. Further, it is important to consider the precise detail provided beneath the headline recommendations. For example, settings should not use Recommendation 5 to justify the purchase

of lots of interventions. Rather, it should provoke thought about the most appropriate interventions to buy.

Inevitably, change takes time, and we recommend taking at least two terms to plan, develop, and pilot strategies on a small scale before rolling out new practices across the setting. Gather support for change across the setting and set aside regular time throughout the year to focus on this project and review progress. You can find out more about implementation in our guidance report [‘Putting Evidence to Work—A School’s Guide to Implementation’](#). Though it is not specific to the early years, the principles will be useful for all settings.

“Implementing the recommendations will require careful consideration for how they fit your setting’s context”



SUMMARY OF RECOMMENDATIONS

Sections are colour coded for ease of reference

Find this info on:

1

Develop practitioners' understanding of how children learn mathematics



- Professional development should be used to raise the quality of practitioner' knowledge of mathematics, of children's mathematical development and of effective mathematical pedagogy.
- Developmental progressions show us how children typically learn mathematical concepts and can inform teaching.
- Practitioners should be aware that developing a secure grasp of early mathematical ideas takes time, and specific skills may emerge in different orders.
- The development of self-regulation and metacognitive skills are linked to successful learning in early mathematics.

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2

Dedicate time for children to learn mathematics and integrate mathematics throughout the day

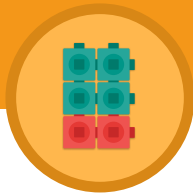


- Dedicate time to focus on mathematics each day.
- Explore mathematics through different contexts, including storybooks, puzzles, songs, rhymes, puppet play, and games.
- Make the most of moments throughout the day to highlight and use mathematics, for example, in daily routines, play activities, and other curriculum areas.
- Seize chances to reinforce mathematical vocabulary.
- Create opportunities for extended discussion of mathematical ideas with children.

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3

Use manipulatives and representations to develop understanding



- Manipulatives and representations can be powerful tools for supporting young children to engage with mathematical ideas.
- Ensure that children understand the links between the manipulatives and the mathematical ideas they represent.
- Ensure that there is a clear rationale for using a particular manipulative or representation to teach a specific mathematical concept.
- Encourage children to represent problems in their own way, for example with drawings and marks.
- Use manipulatives and representations to encourage discussion about mathematics.
- Encourage children to use their fingers—an important manipulative for children.

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4

Ensure that teaching builds on what children already know



- It is important to assess what children do, and do not, know in order to extend learning for all children.
- A variety of methods should be used to assess children's mathematical understanding, and practitioners should check what children know in a variety of contexts.
- Carefully listen to children's responses and consider the right questions to ask to reveal understanding.
- Information collected should be used to inform next steps for teaching. Developmental progressions can be useful in informing decisions around what a child should learn next.

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5

Use high quality targeted support to help all children learn mathematics



- High quality targeted support can provide effective extra support for children.
- Small-group support is more likely to be effective when:
 - a. children with the greatest needs are supported by the most experienced staff;
 - b. training, support and resources are provided for staff using targeted activities;
 - c. sessions are brief and regular; and
 - d. explicit connections are made between targeted support and everyday activities or teaching.
- Using an approach or programme that is evidence-based and has been independently evaluated is a good starting point.

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1 Develop practitioners' understanding of how children learn mathematics



Early years settings and schools should invest in developing practitioners' own understanding of mathematics, their understanding of how children typically learn, and how this relates to effective pedagogy.⁵ This is important for realising the potential of the other recommendations in this guidance report.

Effective mathematics teaching requires knowledge of mathematics pedagogy and learning as well as of mathematics itself. This includes knowledge of how children learn mathematical concepts, connections between mathematical concepts, likely difficulties children may have, and different approaches to solving problems or tasks. Professional development should therefore focus on the integration of three areas: mathematics itself, children's mathematical development, and of effective mathematical pedagogy.⁶

How do children typically learn mathematics?

In order to lay the foundations for learning, settings should embed practices that support relationships with children and extend children's learning.⁷ This includes:

- staff knowing individual children well;
- a culture that supports children's curiosity, thinking, and problem-solving; and
- children engaging in appropriate, cognitively-challenging activities.⁸

Mathematical development involves acquiring skills, conceptual understanding, and factual knowledge across a range of topic areas, including quantity and number, operations, shape, and space. It involves forming connections between concepts, such as understanding that addition is the inverse of subtraction. Children also need to develop reasoning skills such as logical thinking and the ability to justify and explain their ideas.⁹

The rate of mathematical development does not rely only on specific mathematical knowledge and skills but also on:

- executive functions such as working memory¹⁰—the ability to hold information in your mind and manipulate it—and self-regulation and metacognition skills (see Box 1);
- language skills (see Recommendation 2)—language skills enable the effective communication of mathematical thinking,¹¹ and a grasp of mathematical language is important for the development of mathematical ideas;
- motor skills—required for a variety of mathematical activities, for example, counting objects and writing numerals.¹²
- children's prior experiences with mathematical materials and activities;¹³ and
- children's interests, enjoyment, and attitudes towards mathematics.¹⁴



Box 1: Developing self-regulation and metacognitive skills

‘Self-regulation’ refers to the ability to understand and manage one’s own emotions, behaviour and thoughts in different situations. To successfully complete a mathematical task, children must be able to self-regulate, and so the development of self-regulation is consistently linked to successful learning in early mathematics.¹⁵

Metacognition is the ability to reflect on your own thinking processes and is closely related to self-regulation. Practitioners should encourage children to explain their thinking processes and strategies when solving mathematical problems. Such monitoring of problem-solving processes enables children to gain insight into their own thinking, learn from their errors, and develop their problem-solving skills.¹⁶

Practitioners can support children to develop these skills by describing the child’s strategies and approaches linked to thinking and learning. For example: ‘I can see that you are thinking really carefully about where the corners are on the jigsaw pieces—that could help you to find the right place for it’, or, ‘I can see you’re finding it difficult to concentrate, we could find a quieter place to work so that we won’t be disturbed. Practitioners may also talk through their own problem-solving strategies out loud whilst solving a problem, to model this thinking to children. This can involve modelling getting stuck and reflecting on different strategies.

The EEF’s guidance report [‘Metacognition and Self-Regulated Learning’](#) provides more detail on these skills and approaches.¹⁷

Developmental progressions

A substantial amount of research on how children learn mathematical concepts has revealed the complexity of mathematical development. Developing a secure grasp of early mathematical ideas takes time. Even if a child appears to be engaging successfully in mathematical activities (for example, reciting the count sequence), they may not have a full grasp of the underlying concepts (for example, the meaning of numbers in the count sequence). Children may also appear to have grasped an idea in one context but then fail to show that knowledge in a different context.¹⁸

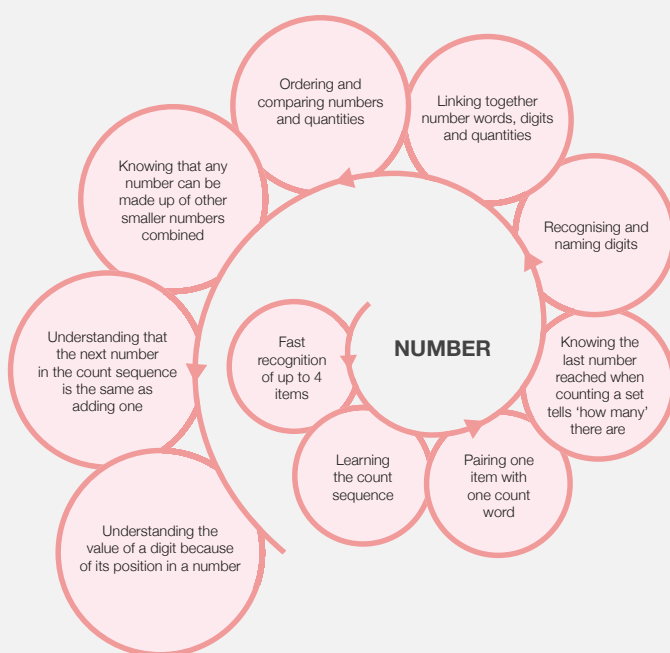
However, research has suggested possible paths that children may follow in developing an understanding of a mathematical topic. ‘Developmental progressions’ are descriptions of the typical path that children tend to follow in developing an understanding of a mathematical topic.

There are several developmental progressions available;^{19 20 21} these vary in their focus and the amount of information they present. The diagrams below provide simple examples of progressions in number development, operation development, and geometry and spatial thinking. The spiral highlights

the progression of individual skills or concepts that develop over time.

The diagrams are a spiral to convey that whilst there is some ordering in which these skills may emerge, development does not take place in clearly defined linear steps. Children may develop several skills in parallel and individual children may move through skills in different orders. However, to reach full understanding, children will need to master each of these skills. Developmental progressions can, therefore, be seen as approximate paths of the development of thinking, but not a clear linear progression for all.

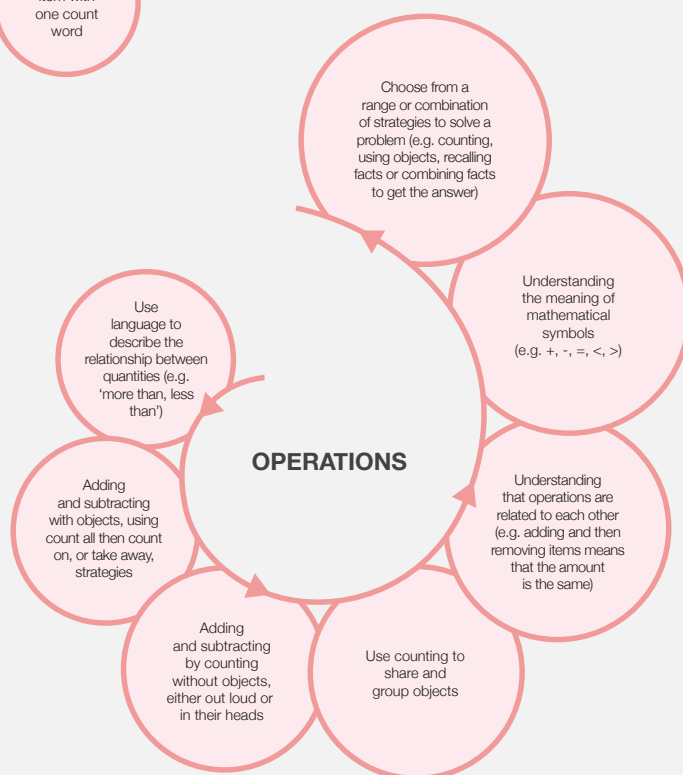
While each spiral is presented as a separate diagram, there is considerable overlap in development across these topics. In particular, understanding of operations builds upon children’s understanding of number. Across all three topic areas, children must come to understand ideas of composition and decomposition—putting together and taking apart—as this is fundamental to both number and arithmetic (for example, part-whole relations, adding and subtracting) and geometry (for example, shape composition).



These diagrams summarise what we know about number and operations development. The development of each skill or concept is not discrete, instead there is considerable overlap in development so children may develop several skills in parallel. Children may also move through the spiral in different orders.

There is considerable overlap in development across these topics. In particular, understanding of operations builds upon children's understanding of number.

This number diagram is based on a large body of research on development and the operations diagram is based on a moderate body of research about development.



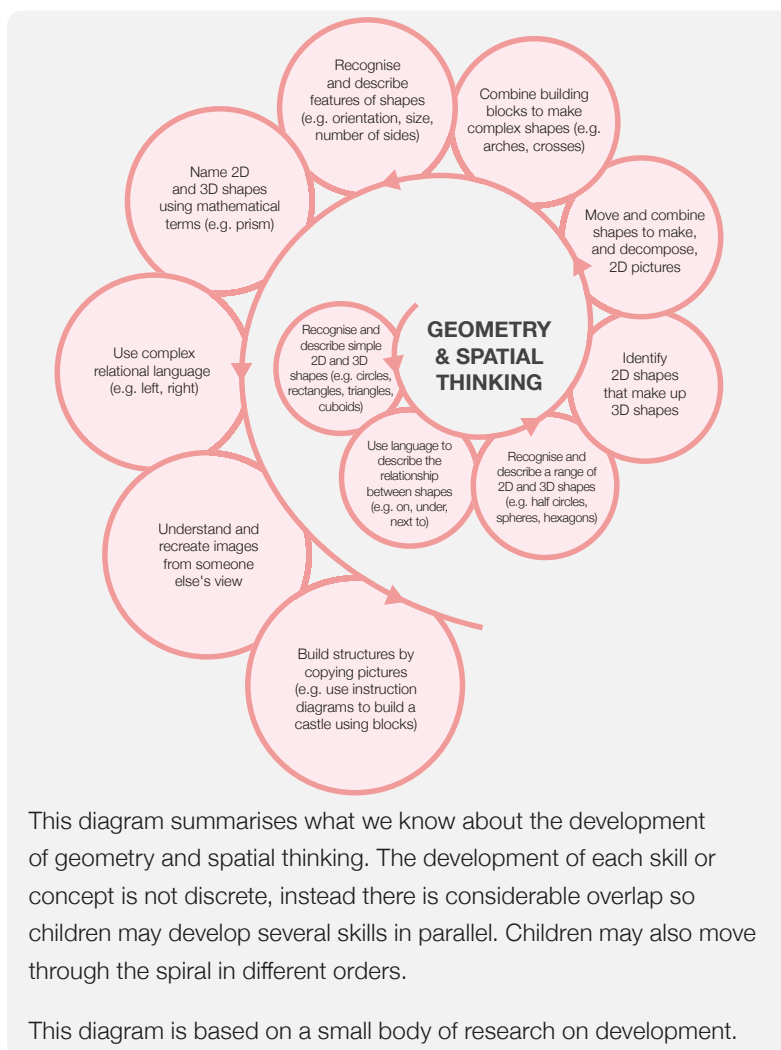


How can I use this information?

It is important that practitioners are aware of typical development of mathematical skills and concepts to inform teaching. This knowledge will support the implementation of the other recommendations in this guidance report.

Such knowledge can support practitioners to:²²

- have a good understanding of what children need to learn to progress;
- make judgements about the range of experiences that children may benefit from to develop a full understanding of mathematical topics;
- determine the developmental pre-requisites for a particular skill;
- assess a child's level of understanding; and
- intervene at the appropriate levels of challenge and build on what children already know.



First stop for further reading

Clements, Douglas H. and Sarama, Julie (2014). *Learning and Teaching Early Math: The Learning Trajectories Approach* (2nd edn) (Studies in Mathematical Thinking and Learning Series, London: Routledge).

Also see <https://www.learningtrajectories.org>.

Researchers Douglas Clements and Julie Sarama have developed detailed developmental progressions that also include activities and tasks for each step to support children to achieve the next level of thinking, which are coined 'Learning Trajectories'.

The Early Math Collaborative, Erikson Institute (2014). *Big Ideas of Early Mathematics; What Teachers of Young Children Need to Know*, Pearson.

This book by the Erikson Institute explores key concepts in early mathematics learning and implications for teaching, including activity ideas.

Gilmore, C., Gobel, S. and Inglis, M. (2018). *An Introduction to Mathematical Cognition*. (London: Routledge).

This book explores the research that seeks to understand how people come to understand mathematical ideas.

2 Dedicate time for children to learn mathematics and integrate mathematics throughout the day



In the early years and Key Stage 1, there are opportunities throughout the day to learn about mathematics through the environment and through routines, as well as in dedicated teaching time. Throughout the day children should be exposed to mathematical language, both formal and informal, and practitioners should discuss mathematical ideas with children in order to extend their thinking.

Plan daily activities targeting specific maths concepts and skills

Practitioners should dedicate time each day for purposeful mathematics activity.²² This should focus on supporting children to develop specific mathematical ideas and skills, taking into account developmental progressions. Practitioners can use whole-class, large and small groups to tailor instruction for children who need support on different aspects of content. Approaches will look different for children at different developmental stages, and in the early years activities are likely to be short and active.

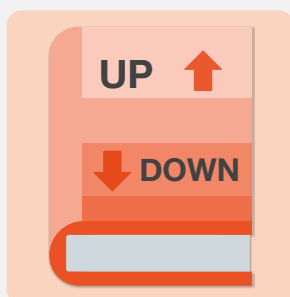
Mathematics can be explored through different contexts, including books, puzzles, songs, rhymes,

puppet play and games. Using storybooks to teach mathematics can be particularly effective, through providing an opportunity for mathematical talk and questioning. Much of this evidence comes from studies where practitioners were explicitly supported in promoting mathematical discussion from the story, for example, by being provided with notecards displaying prompting questions and discussion points that they could use.²³ Practitioners should therefore plan how they will use storybook resources to discuss mathematical concepts.



Box 2: Using storybooks

There are a number of mathematics story and picture books available across a range of ages that can be powerful in engaging children with mathematical concepts. *Development and Research in Early Math Education* (DREME), an early mathematics research network based at Stanford University, provides evidence-informed guidance for practitioners and parents on choosing books with appropriate mathematical content. It produces 'Storybook Guides' to support effective use of the books, including ways to maximise the mathematical talk.



Extract from resource for the storybook 'Inside, Outside, Upside Down' by Stan and Jan Berenstain (1996). Suitable for ages 2–4.

A bear gets in a box that gets turned upside down, taken outside, and put on a truck. Children learn about spatial words.



Extract from resource for the storybook 'One is a Snail, Ten is a Crab' by April and Jeff Sayre and Randy Cecil (2003). Suitable for ages 5–8.

Each page invites children to count the number of feet that people and different animals have.

Mathematical talk during reading:

- Talk about what is happening in the picture, emphasising words that describe spatial relationships—*Do you see Brother Bear getting in the box? Where is the box going? How do you know?*
- Consider opposites—*What is the opposite of going on the truck? What is the opposite of getting inside the box?*
- Make predictions—*What might happen next in the story? If the box is upside down and we turn it around, will it still be upside down?*

Mathematical talk during reading:

- Explore counting the number of feet in different animals—*How many feet does an insect have? Can you show me with your fingers?*
- Practise adding one more—*What happens when we add one foot? How many feet do we have altogether? How do you know?*
- Explore counting by groups of 10—*If we have two crabs, how many feet do they have altogether? How do you know?*

From [Development and Research in Early Math Education](#).

[Mathematics Through Stories](#), a U.K. organisation that promotes the teaching of mathematics through stories, is another useful source for stories and resources.

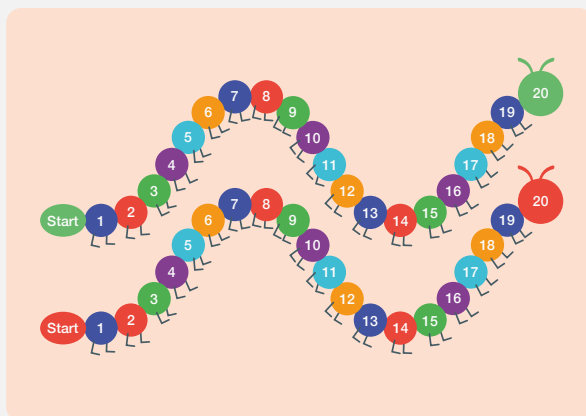
Games can be an engaging way to practise and extend skills. They can build on children's mathematics knowledge, generate repeated practice in a motivating context, and give children and practitioners an opportunity to discuss strategies and ideas. Practitioners should select specific games to suit current objectives to provide appropriate challenge.

There is some evidence that board games with linearly arranged, consecutively numbered, and equal-sized

spaces may be particularly beneficial to numerical understanding, by providing opportunities for developing strategies such as 'counting on'.²⁴ Snakes and Ladders is an example of a commercial board game that may support this, or practitioners may produce their own versions based on the research. An example from the Mathematical Reasoning project, evaluated by the EEF,²⁵ is the caterpillar game (see Box 3).

Box 3: EEF Promising Project—Mathematical Reasoning

Mathematical Reasoning is a whole-class programme designed to develop pupils' understanding of number and reasoning about quantity. The focus of the programme is on problem-solving, reasoning, and understanding, and concepts include the additive composition of number (the principle that numbers are composed of other smaller numbers combined), the inverse relation between addition and subtraction, and reasoning about multiplication and division. Each new concept is introduced through teacher-led activities during which the children use manipulatives to enact a story problem. The programme also includes consolidation through games.



Example activity: Caterpillar game

In this whole-class activity, the teacher projects the slide on the board and the class plays as two teams racing to reach the end of the caterpillar. Each team throws the dice and pupils work out where their mark will be by counting on from where the marker is. In order to finish and win the game, the team must end exactly on the last square, requiring them to anticipate what number they need to finish and compare this with the number thrown.

The EEF conducted a randomised controlled trial of Mathematical Reasoning in Year 2 with 55 schools,²⁶ where teachers were trained in the approach and provided with classroom exercises and training materials. Children who experienced the Mathematical Reasoning approach made an additional three months' progress in mathematics compared to the control group. The EEF then funded a scaled-up evaluation²⁵ in partnership with the National Centre for Excellence in Teaching Mathematics. In this second, larger trial, pupils who experienced Mathematical Reasoning made one month of additional progress compared to the control group. Mathematical Reasoning is an EEF Promising Project.

Reinforce mathematical vocabulary and create opportunities for discussion of mathematics

Practitioners should seize chances to reinforce mathematical vocabulary—for example by making a comment about which child is standing ‘first’, ‘second’, or ‘fourth’ in line, which child has ‘more than’ or ‘fewer’ objects than another child, or helping children rephrase statements that use ambiguous, non-mathematical language, such as refining ‘big’ when the child means ‘tall’.

It is important that children are supported to use informal language to describe mathematical ideas for example ‘more than’, ‘smaller than’, ‘pointy’, ‘curved’. Once children are comfortable with using informal language, practitioners can introduce more formal mathematical vocabulary.²² Practitioners need to consider how formal vocabulary is introduced. For example, it may be more beneficial to introduce formal names of shapes gradually rather than all at once then reinforce this as part of an ongoing routine. Practitioners in a setting could plan their use of mathematical language to ensure a consistent approach.

Practitioners should create opportunities for extended discussion of mathematical ideas with individuals or small groups of children in order to extend their thinking. This can be particularly effective when a child is showing an interest in a certain problem or activity. A number of different frameworks exist to support high quality interactions, such as guided interaction²⁷ and sustained shared thinking.²⁸

Being highly tuned-in to the child’s behaviour and motivations, responsive to what children are saying and using a variety of techniques to help develop and extend children’s thinking are central to these approaches, which can be used while children engage in a variety of everyday activities.

Features of such approaches include:

- the use of *open-ended questions*: ‘How did you...?’, ‘Why does this...?’;
- asking children to *elaborate*: ‘I really want to know more about this...’;
- *recapping*: ‘So you think that...’; and
- *clarifying ideas*: ‘So you think we should...?’.

“Create opportunities for extended discussion of mathematical ideas with individuals or small groups of children”

Highlight mathematics across the day

Throughout the day there will be meaningful ways to use mathematics. Mathematics can be highlighted through daily routines, during play, and in other curriculum areas.

Everyday routines such as registration time, snack time, and tidying up provide opportunities for counting

and comparison as well as addition, subtraction, sharing, and time problems. Practitioners should take advantage of such time to support mathematical development, for example, by engaging in mathematical conversations, singing a song, or playing a number game.

Box 4: Using routines to practise mathematics



Practitioners in a nursery class decided to use snack time to help the children to recognise numbers of objects and connect them to number words. During snack time they would point out, 'We have three oranges that we are going to share out', whilst showing the numbers on their fingers to reinforce the quantity. By encouraging the children to see the amount of something rather than only seeing the object—for example, the orange or cup—the practitioners were helping the children to develop the habit of quantifying small groups or collections. This enabled children to begin to build up a concept of number and connect number words with amounts.

A Year 1 team looked at the learning objectives for their year group and identified those which they thought could be effectively consolidated through classroom routines. One of these objectives was counting to 100 forwards and backwards. The teacher decided that every day when two children went to get the fruit, the others would count how long it took them. For the first few months they counted forwards. Sometimes they would just say the number names. On other days the teacher would also point to the numbers either on a 100 square or on a 0–100 number line. Later in the academic year, the class started counting backwards from 100.

The teacher was aware of typical challenges, such as saying 1 less than a multiple of 10 when counting backwards, so emphasised these numbers in her own counting. By using everyday routines in the school day, this provided additional opportunities for the children to develop fluency in important mathematical skills, and for the team to monitor the development of these.

Practitioners should provide a variety of tools to allow children to explore all areas of mathematics, and opportunities for outdoor provision should be maximised, where this is possible, for the development and reinforcement of mathematical ideas. Appropriate tools include manipulatives, measuring items, scales, construction materials, puzzles, sorting and pattern materials.²⁹ Building blocks may be especially important for developing children's spatial awareness and knowledge of shapes.²¹ Children should have experiences with a wide variety of shapes. For example, exposure to a variety of triangles, rather than limited to certain types, such as equilateral or isosceles.²¹

Practitioners have an important role in scaffolding opportunities for learning and extending the learning during play. Through observing children's play, practitioners will identify 'teachable moments' in which they can join the play to add to the discussion, reinforce mathematical vocabulary, and encourage problem-solving. Practitioners may find it useful to think about mathematics concepts, discussion points, and vocabulary related to the different play areas and activities so they can use them when appropriate moments arise.³⁰

Box 5: Maths Champions, a whole setting approach to improving mathematics provision



Maths Champions is a programme to support nurseries to improve their mathematics provision run by the National Day Nurseries Association (NDNA). The programme involves nursery settings allocating a senior member of staff to take the role of 'Mathematics Champion', responsible for raising the quality of mathematics provision in their setting. The Mathematics Champion receives online training on developing their own understanding of mathematical development, evaluating practice in their setting, and supporting other practitioners' development.

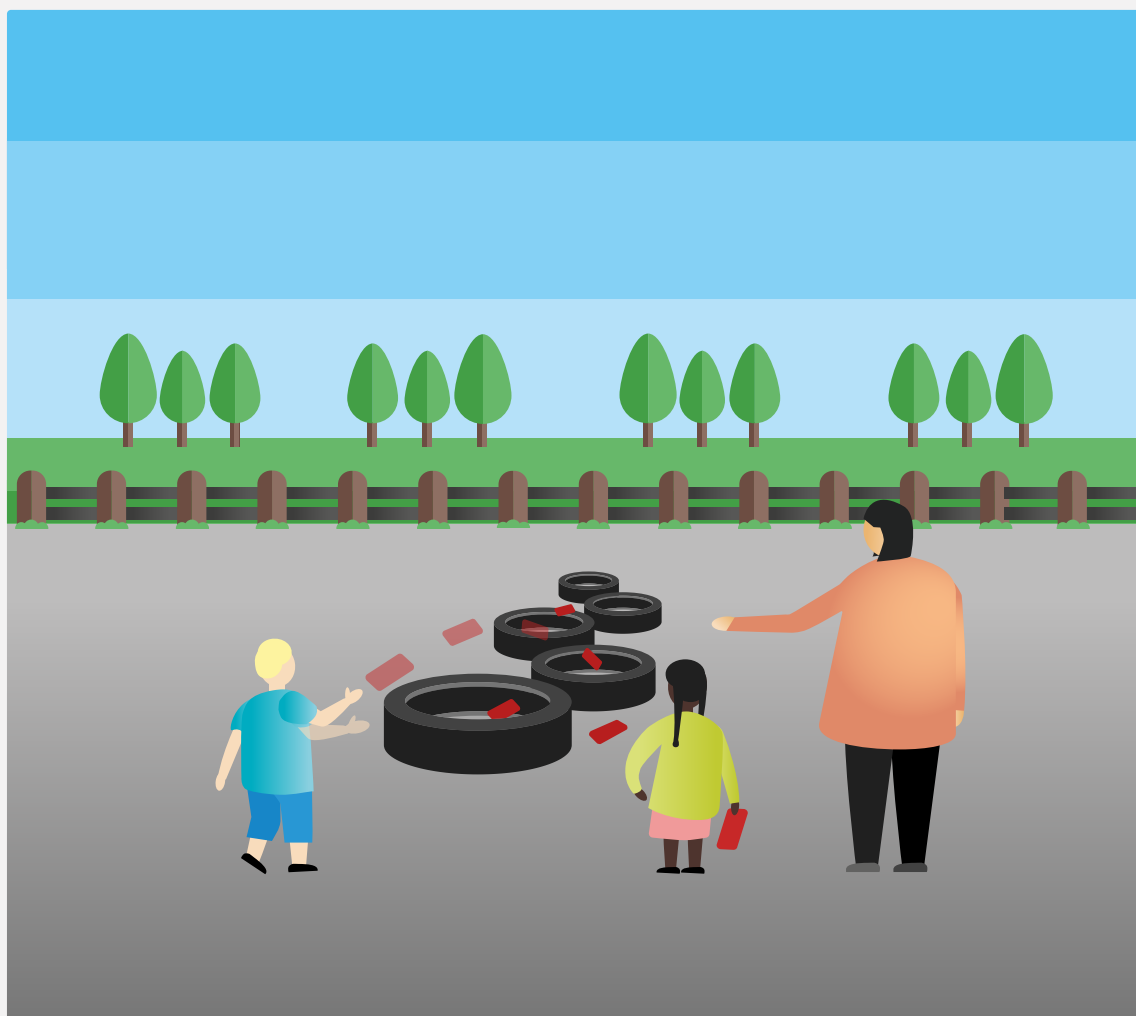
Areas of focus include improving the mathematical environment, for example, the quality of resources available in the nursery to support mathematical learning, encouraging staff to use appropriate mathematical language in discussions with children, and supporting staff to plan opportunities for mathematical learning in play and activities.

An EEF trial evaluated the programme, which focused on children who were three years old at the start of the programme. After one year, children from nursery settings receiving Mathematics Champions made two months' additional progress compared to business as usual control settings and there was suggestive evidence that the quality of maths provision in the participating nurseries was improved.³¹

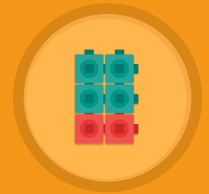
Box 6: Highlighting mathematics during play

During outdoor play, two four-year-old children were taking turns to throw five beanbags into the nearest of a long line of tyres. The teaching assistant encouraged them to say how many they got in the tyre each time and how many were out, and to check that there were always five altogether.

She then set up a whiteboard so they could record their names and scores, either by making marks or by referring to the number track on the fence if they wanted to write numerals. In this way, the children were engaging with counting and values for number symbols, and beginning to see that five could be made up of smaller numbers. The teaching assistant then asked, 'Can you get any beanbags in the next tyre? It's a bit further away!' They discussed how far the children could throw the beanbags and how many they got in the further tyres, referring to them as 'first', 'second', and 'third'. The children were then beginning to use appropriate vocabulary to compare distances and describe numbers ordinally.



3 Use manipulatives and representations to develop understanding



What are manipulatives and representations?

A **manipulative** is an object that children or practitioners can interact with and move to represent mathematical ideas. Manipulatives could include everyday objects such as pine cones, buttons, and small toys as well as resources like interlocking cubes, Cuisenaire rods, Dienes blocks, and building blocks.

A **'representation'** refers to a particular form in which mathematics is presented.³² Representations include informal drawings, mathematical symbols, and more formal diagrams, such as a number line or graph.

Manipulatives and representations can be powerful tools for supporting young children to engage with ideas across many areas of mathematics. They can help children make sense of mathematical concepts, develop visual images, increase engagement and enjoyment, help practitioners see what children understand and provide a bridge to abstract thinking.³³ Children benefit from practical, first hand experiences of moving and interacting with manipulatives to develop mathematical ideas.

It is important that children have opportunities to engage in both free and structured play with manipulatives. However, practitioners must help children to understand the links between the

manipulatives or representations and the mathematical ideas they represent.³⁴ As children's understanding of mathematical ideas develops, practitioners should encourage children to use pictures, symbols and more abstract diagrams to represent and communicate ideas and concepts.

There is some evidence that physical whole-body movement and gestures may support the learning of mathematics, for example, moving along a physical number line,³⁵ or jumping and clapping while counting. Practitioners should encourage children's use of fingers, which can be important manipulatives for children.²⁹ Fingers can be useful for supporting counting and later on for counting in groups.



What does effective practice look like?

The evidence suggests some key considerations:

- **Ensure that children understand the links between the manipulatives and the mathematical ideas they represent.**³⁴ Children need support in linking a manipulative with the mathematical ideas it represents. For example, a child may be confident using Dienes blocks to add but be unable to connect this to a written addition. This requires practitioners to explicitly help children to link the materials (and the actions performed on or with them) to the mathematics of the situation. This should enable children to develop related mathematical images, representations, and symbols.
- **Ensure that there is a clear rationale** for using a particular manipulative or representation to teach a specific mathematical concept. Practitioners should consider carefully how the manipulative will be used to build on existing understanding, and help develop increasingly sophisticated approaches and ideas.²¹
- **Encourage children to represent problems in their own way.**³⁶ Practitioners should support children to become familiar with a repertoire of strategies to use to represent mathematical ideas, including their fingers, drawings, and marks such as tallies and arrows. Children should be free to invent and explore their own representations to record their thinking and communicate their understanding.
- **Be aware that young children can be distracted by the surface features of a novelty manipulative**—this can take away from the intended learning aim. Using a given manipulative regularly, or introducing it through play to gain familiarity can be beneficial.³³
- **Use manipulatives and representations to encourage discussion about mathematics.**³³ Children can work in pairs and small groups using manipulatives to solve problems and to encourage questions about other children's strategies and reasoning. This can prompt the sharing and comparison of different approaches. Manipulatives can also be used by children to communicate what they know.

There is evidence of the importance of showing children different representations of number and then helping them to make connections between them in order to support a fuller understanding.⁶ For example, understanding that the numeral '3', a three on a dice face, three cubes, and a three-step on a number line all represent aspects of 'three'.

It is likely that practitioners' understanding of mathematical concepts needs to be strong in order to use manipulatives and representations effectively, and this may need to be a focus of CPD for some practitioners.³⁷ Settings should also plan their use of manipulatives and representations to ensure a consistent approach.

“Children need support in linking a manipulative with the mathematical idea it represents”

Box 8: Using manipulatives to explore 'one more than'

A Reception practitioner had recently watched 'Numberblocks' (CBeebies) with the children. In this particular episode (season 3 episode 28), the number 15 is represented with interlocking cubes in a staircase pattern.

The practitioner wanted to further explore the 'one more than' relationship between counting numbers with the children. In order to do this children need to know the cardinal value of the numbers (that the last number counted represents the overall amount in the group) and realise that adding one to any number produces the next counting number.

The practitioner showed the children how to build 'staircases', making each 'stair' by matching the previous one, then adding one. The practitioner encouraged the children to use large bricks to make staircases, which were easy for them to handle, and encouraged physical movement of the manipulatives.



In order to support the children to understand the link between the pattern and the 'one more than' relationship, the practitioner started to make a spectacular giant 'staircase' in the playground with some cable spools. This also presented the same pattern in a different context, helping children to recognise the structure in a different orientation and with circular objects.

The practitioner modelled the process of making the staircase, saying, 'How many do I put next?' Sometimes he did it wrong deliberately and matched the previous stair. When a child said, 'You need another one', he said 'Oh, yes, it's got to be one more than the one before', modelling the language to describe the relationship. When it was finished he asked, 'What do you notice?' One child said, 'It's a pattern. It goes up one, one, one.' One child counted the stair columns, saying 'one, two, three, four, five'. Another child standing at the side said they could also count 'five, four, three, two, one', pointing to the rows. This showed that children were becoming aware of different features of the pattern.

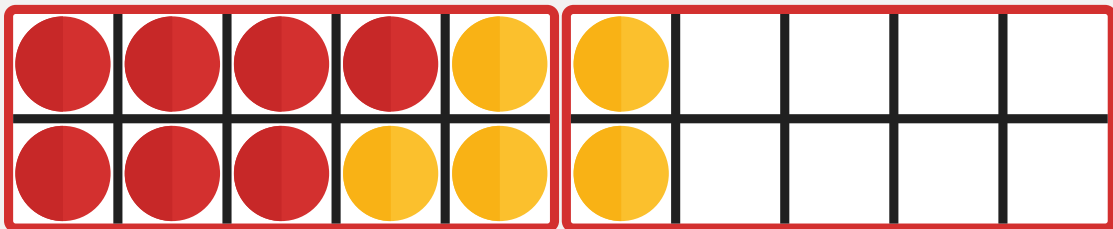
The practitioner later made a staircase with sticks of interlocking cubes wearing number 'hats' to explicitly show the continuing pattern with numerals.

In creating and playing with physical and symbolic representations of the staircase, and discussing 'wrong' or muddled examples, the children were repeatedly meeting the same pattern and becoming familiar with ideas such as the inverse relationship between 'one more' and 'one less' and of hierarchical inclusion—that each successive number is equal to the previous number plus one.



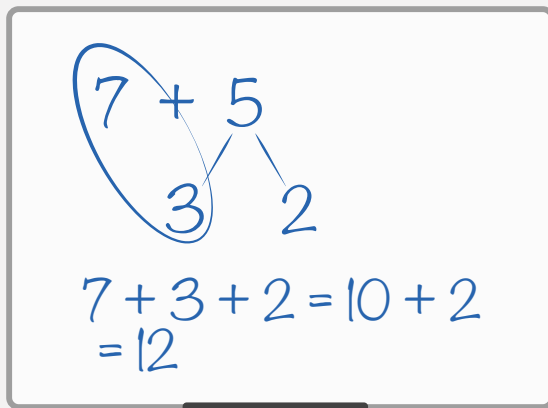
Box 9: Using manipulatives to explore bridging through ten

A Year 1 teacher wanted to teach children to use their number bonds to add numbers by bridging through ten (adding two numbers whose total is greater than ten by counting through to ten then adding the remainder). Using the example of $7 + 5$, she chose to use double sided counters and ten-frames and demonstrated putting seven red counters on the frame, then adding five, yellow side up. This allowed the children to focus on splitting the five to into three (to make ten) and two more, with the ten-frame clearly showing the resulting 12 as ten and two. For children who found it difficult to keep the counters in place on the ten-frames she provided chunky 'jewel' counters and ice cube trays in tens.



The teacher then modelled this in abstract form alongside the ten-frame.

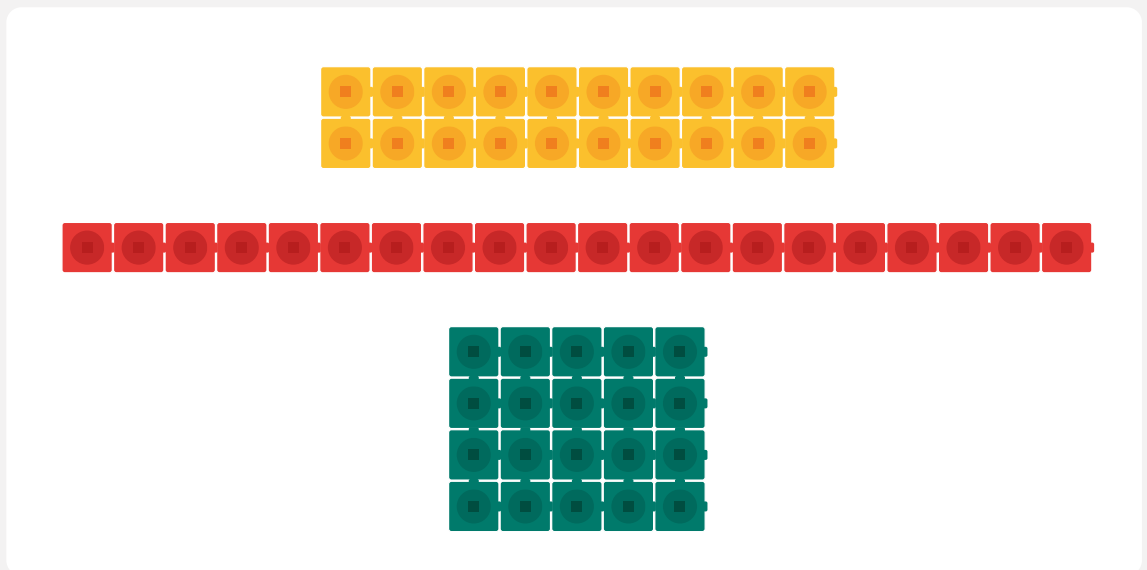
This helped children to represent this concept abstractly and make the explicit link between the manipulative and the idea it represents. The children then played a game in pairs, using a dice numbered five to ten to decide the first number, then adding five.



Box 10: Using manipulatives to explore making rectangles

A Year 2 teacher wanted to introduce children to the multiplicative composition of numbers, for example that 8 can be expressed as two fours and four twos, and to increase familiarity with some key multiplication facts.

He challenged them to make as many different rectangles as they could with 20 cubes. This enabled them to see that 20 could be made with two lots of ten or four lots of five. He then asked them how they knew they had found all the arrays for 20 and to check. Most children worked randomly, trying different numbers in rows. One child checked all the numbers from 1 and then discovered the 1×20 array. The teacher chose multilink cubes because these could be joined to form arrays, which could be turned round to clearly show, for instance, that two rows of ten are equivalent to ten rows of two. This demonstrates the commutativity of multiplication (that the order of multiplying two numbers does not affect the result).



He asked the children to describe the five by four array and to say what they noticed when they turned the array round. One child said, 'I can see five and five and five and five, and when I turn it round I can see four and four and four and four and four'. Other children said things like, 'It's five rows of four this way and four lots of five the other way' and 'It's five fours and four fives'. This showed the teacher that the first child was thinking additively, whereas children who could talk about 'lots of', 'rows' or 'four fives' were thinking multiplicatively. When he asked them to record all the ways they had found, this similarly showed their understanding, as some wrote $5 + 5 + 5 + 5$ while others wrote 5×4 . In this activity the manipulatives supported the children to articulate their reasoning and helped the teacher to identify children beginning to work multiplicatively.

Activity adapted from 'Making Numbers: Using Manipulatives to Teach Arithmetic', Oxford University Press.



For more examples of using manipulatives:

Griffiths, R., Gifford, S., and Back, J. (2016). *Making Numbers: Using Manipulatives to Teach Arithmetic*. Oxford University Press.

The Nuffield Foundation commissioned a literature review on the use of manipulatives for teaching arithmetic to children. This made recommendations for practitioners, which can be found at: <https://www.nuffieldfoundation.org/using-manipulatives-foundations-arithmetic>. Making Numbers was a book produced as part of this project to provide exemplification of using manipulatives.

4 Ensure that teaching builds on what children already know



Children enter each educational phase at different starting points, as some children will not have had the variety and amount of mathematical experiences that others have. Practitioners should, therefore, carefully assess what children do and do not know in order to identify the appropriate areas to extend learning. Information can be used by practitioners to inform how to give feedback, intervene, and plan future activities, so as to support children to develop more sophisticated mathematical concepts and skills.

Knowledge of the development of mathematical thinking is required to interpret and use observation to inform teaching.²¹ Knowledge of developmental progressions (see Recommendation 1) can inform decisions about what questions to ask to assess understanding, what a child should learn next and then how to choose activities that are at or slightly above a child's level of understanding.²²

As early mathematical concepts take time to develop, observing a child demonstrating a mathematical skill does not necessarily mean that understanding is secure. It often takes time to consolidate learning and transfer that learning to different contexts, so practitioners

should ensure they check what children know in a variety of contexts.

A variety of methods should be used to assess children's mathematical understanding. This may include observation of children while engaged in activities, setting specific tasks to reveal understanding and discussions with children about mathematics and their reasoning. Practitioners could target specific assessment activities to children they have a clear rationale for focusing on instead of giving a blanket assessment to all children. The choice of strategy should be age-appropriate; written, timed assessments will not be appropriate for younger children.

Take opportunities to gather information about what children know

Observing children provides the opportunity to see what learning children can apply independently. There are many opportunities throughout the day to observe what children do and say that reveals their mathematical knowledge and understanding. For example, noticing the way a child engages in block play, plays a game, or counts objects will give an indication of their developing mathematical knowledge. This provides opportunities for practitioners to discuss mathematics with children to find out more about their understanding, intervene to further develop mathematical thinking, and use the information to plan future activities.

When discussing mathematics with children, practitioners should listen carefully to children's responses and explanations, rather than focus on a particular answer to a problem, and consider good questions to ask to reveal understanding.²¹ Open ended questions such as 'How did you know?' and 'What did you do first?' can inform appropriate feedback, teaching and support. This is especially important for errors or partial understanding; it is important that the practitioner explores why the error occurred in order to reveal the level of thinking. This will help shape an appropriate response to support children to improve their mathematical knowledge and understanding.

Plan specific tasks to gather information about what children know

Carefully designed tasks can reveal children's understanding, which can be used by practitioners to support learning. When planning a task or activity to

collect information, it is important for practitioners to plan exactly what information they want to get out of the activity and what they will do with information.

Box 11: Taking opportunities to gather information about what children know

In Reception, Mustafa and Rohan were putting construction blocks away by matching them to the shape silhouettes on the shelves. Mustafa looked hard for the outline for a semi-circle before finding it and slotting the shape in. 'Well done', said the teacher, 'that was a tricky one! How did you know it went there?' 'It's round and straight', said Mustafa, 'and two of them make a circle'. Then Mustafa stood looking to see where he could put a triangular block. The teacher noticed he was holding it the other way up to the shape outline. 'What shape are you looking for?' she asked. 'One with points,' said Mustafa and then spotted the outline but put the shape the wrong way up on it. 'Can you turn it round so it fits?' she asked, and Mustafa successfully turned it round and put it on the outline. His friend Rohan said, 'And two triangles fit on top of the brick ones' and put two right-angled triangular blocks together to make a cuboid.



The teacher noted that both children could use informal shape vocabulary successfully and that this helped Mustafa to focus on key properties. He was also able to rotate a shape to fit an outline when prompted; Rohan was more confident with this. Both children were beginning to compose shapes from other shapes, so vocabulary, rotation, and composition were all areas for development.



The teacher decided to plan some group challenges with pattern blocks and shape puzzles. These would involve discussing the properties of shapes needed to fit in gaps, encouraging children to predict which shapes would fit if rotated and which shapes would be made by combining them. This could be followed by free play and a range of shape puzzles to solve.

Box 12: Planning specific tasks to gather information about what children know

A teacher was assessing whether his Year 2 class had understood that in order to add 10 to a number we can think about adding 1 ten rather than 10 ones.

He knew that posing questions such as $27 + 10$ or $10 + 35$ in a written assessment wouldn't give him the information he needed as he wouldn't be able to tell whether children had counted on their fingers in ones to solve the calculation or not. Instead he decided to take a small group of children at a time and ask them to work out questions involving adding 10 on mini-whiteboards.



By watching the children within each group answer the questions he was able to assess who could quickly answer it and who was counting on their fingers. Where he wasn't sure which approach a child had used, he asked them to explain their thinking. It took him 15 minutes to assess the whole class in small groups, and from this he was able to identify who had not yet grasped the important concept of adding 1 ten. He planned additional teaching for these children.

Assessing what children know about counting

There are some well researched common errors that children typically tend to make when learning to count. Common counting errors include counting an object twice, skipping over an object, skipping number words or saying number words in a different order.

It can be helpful for practitioners to make statements of various aspects of counting; for example, 'Remember that each object needs one point and one number word', 'You can't skip any', or 'Remember where you started in the circle so you stop just before that'. However, as counting requires effort and continued attention, it is normal for young children to make occasional errors, especially on larger sets. It is more important for children to be enthusiastic counters who enjoy counting rather

than them worrying so much about errors that they are reluctant to count.²¹ Therefore sometimes it is appropriate to let errors go, if children are trying hard, as long as children understand that correct counting requires one point and one word for each object and are trying to do that—and are given engaging further practice opportunities.

An effective method of checking children's understanding of counting can be to ask a child to judge if a puppet is counting correctly. Under these conditions, children can often detect counting errors better than they can perform the counting task themselves.³⁸ Puppets can be used in this way by practitioners to find out the specifics of what children know.

Box 13: Using puppets to gather information about what children know



A nursery practitioner wanted to find out whether a child understood the cardinal counting principle (that the last number counted represents the overall amount in the group). She acted out asking a teddy to give her five cakes and the teddy counting out four cakes, saying 'one, two, three, four', then declaring 'there you are—five cakes'.

The child corrected the teddy, saying, 'No, there are only four cakes!' This suggested to the practitioner that the child had a relatively good grasp of the cardinal principle.

The practitioner then wanted to see if the child could compare numbers and understand their cardinal value as quantities. Together they acted out the teddy sharing seven cakes between himself and another teddy, so he had four and the other had three. 'There,' said the teddy, 'that's fair, we both have the same.' The child said: 'No! That's not fair—you've got four and the other teddy's only got three!'

This suggested to the practitioner that the child could compare the number words, 'three' and 'four'. The child understood the cardinal value of the numbers, probably through subitising (recognizing the number by the visual arrangement, without counting). The child did not demonstrate the comparative language, so the practitioner modelled this by saying, 'Teddy, we think that you've got more, because four is more than three.'

She planned for the child to join others playing the 'Who has most?' game, where two toy animals take it in turns to throw a dice and receives that number of counters. The practitioner then asks 'Who has more?' and 'How do you know?', encouraging the children to say, for instance, 'The bunny has the most, because six counters is more than four counters.'

'Who has most?' activity adapted from 'Making Numbers: Using Manipulatives to Teach Arithmetic', Oxford University Press.



Box 14: A whole-setting approach to understanding what children know



A nursery school reviewed its mathematics provision for three- and four-year olds. One of the findings of the review was that when practitioners counted along with groups of children, it was often difficult to tell if individual children knew or did not know the correct number order. Some children appeared to be counting along correctly, but it was hard to hear exactly what they were saying because this was drowned out by the other children's louder voices. It was also difficult to tell whether every child knew that the last number they counted represented the number of objects in the set.

Overall, practitioners were not always clear about what children knew and could do.

So, the team piloted a different approach to checking children's understanding of number. When counting, they stopped leading groups in chanting to large numbers like 20. Instead, they focused on small sets of up to five. Individual children were asked to say how many items were in the set. Some children could tell just by looking (subitising) and others needed to count. In a few cases, children did not know the correct sequence of numbers. Practitioners gave those children extra help to catch up using engaging approaches like finger rhymes with counting.

Children were also given extra opportunities to deepen their understanding of small numbers. For example, at snack time a practitioner asked a child to 'get everyone a plate, please'. There were four children, but the practitioner was careful not to ask for 'four plates'. She wanted to use this as an opportunity to find out what the child knew and could do.

Pen and pencil pots were reorganised with just five items in each pot, so children practised counting up to five every tidy-up time. The overall approach involved much more open-ended and practical exploration of numbers, with children being asked questions like, 'How did you know there were four there?' Where children clearly showed that they knew and could use numbers up to five in a range of contexts, practitioners encouraged them to explore part-whole relationships, like finding lots of different ways of building with five wooden blocks. They continued to engage with children's interest in larger numbers, too, but moving the children onto bigger numbers was only prioritised once they achieved a deep understanding of smaller numbers, like 'the fiveness of five'.

5 Use high quality targeted support to help all children learn maths



Settings should focus on improvements to mathematics planning and pedagogy that support all children. With this in place, the need for extra support for children should reduce. However, sometimes children will need targeted small-group or one-to-one support and attention to continue to make progress in mathematics. This is especially the case in this phase as children enter with a wide range of prior experiences of mathematics.

Support should be informed by a good understanding of what children do and do not know and focus on what they need to learn in order to make progress (see Recommendation 4). Many children who may need additional support with early mathematics can return to a typical learning trajectory with varied teaching methods and targeted individual support.²⁹

Much of the evidence is based on structured interventions developed by expert teams that have been informed by research on children's mathematical development, which include training and pre-specified materials.

Consider the following to make the most of targeted support:

- use more experienced staff to support children with the greatest needs;
- provide training and support for staff using targeted activities, including structured resources or activity plans with clear objectives;
- sessions should be brief and regular;
- quality is generally more important than quantity—there is some evidence to suggest that time-limited interventions may be more effective;³⁹ and
- make explicit connections between targeted support and everyday activities or teaching; practitioners delivering additional support should have time to discuss this work with their colleagues.



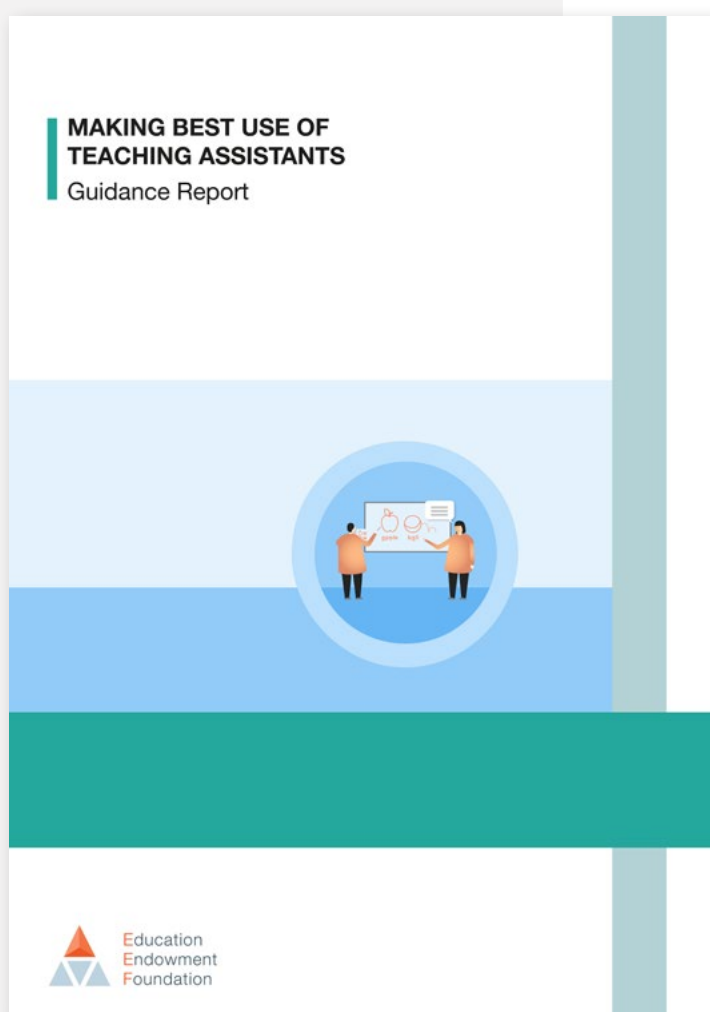
Box 15: Who should deliver targeted support?

It is important that all staff are aware of the needs of individual children and there is shared responsibility among the staff team for supporting children to make progress. If all practitioners know what specific areas of challenge a child may have, they can collectively build the level of support the child has in that area.

The evidence suggests that interventions delivered by teaching assistants (TAs) can have a positive impact on learning, but, crucially, these positive effects occur when TAs work in structured settings with high quality support and training. When TAs are deployed in more informal, unsupported instructional roles they may have little impact or even affect children's learning outcomes negatively. In other words, what matters most is not whether TAs are delivering interventions but how they are doing so. In this context, structured evidence-based programmes provide an excellent means of aiding high-quality delivery. Evidence has shown that with high quality structured programmes involving training, those delivered by TAs can be as effective as those delivered by teachers.⁴⁰

The EEF report [Making Best Use of Teaching Assistants](#) provides more guidance.⁴¹

“High quality structured programmes delivered by TAs can be as effective as those delivered by teachers”



There is evidence that interventions delivered through computers or apps can have a positive effect on children's attainment in mathematics. Such interventions typically provide a structured environment for learning and practice and often include corrective feedback. Some are in the context of a specifically designed game. Initial adult support is often required to enable children to use the software well. However, it is important to note that simply delivering an intervention through a computer doesn't make it effective; the intervention must be based on strong pedagogy. The EEF's *'Using Digital Technology to Improve Learning'* report provides more guidance on the use of digital technology.⁴²

A number of structured intervention programmes, typically comprising training and supporting resources, are available to support the teaching of mathematics in early years and primary settings. Many of these programmes claim to be supported by evidence but it can be challenging to assess these claims or make comparisons between programmes. Resources like the EEF's Promising Projects list and the Evidence 4 Impact database provide guidance on the existing evidence for different programmes.^{43 44} Using a programme that has been evaluated and found, on average, to be effective is a good starting point, but considering the issues above will help to maximise the impact of any programme.

Box 16: 1stclass@number—High-quality structured Teaching Assistant support



1stClass@Number was developed by the Every Child Counts team at Edge Hill University to support pupils who are struggling with mathematics. The EEF conducted a randomised controlled trial of the approach in Year 2 with 133 schools.⁴⁵ The team trained TAs to deliver highly scripted lessons to small groups of up to four children. The programme was implemented outside of mathematics lessons and covered five basic mathematics topics: the number system, place value, addition, subtraction, and multiplication. The lessons also included the teaching of procedures, mathematical signs and mathematical language and concepts. Schools delivered 30 lessons of approximately half an hour, three times a week for ten weeks. A classroom teacher colleague was expected to meet the TA once a week to help them review and plan upcoming lessons, and to provide feedback.

Pupils who received 1stClass@Number made two months' additional progress in mathematics, on average, compared to pupils in the control group. 1stClass@Number is an EEF Promising Project.



Box 17: onebillion—An app based intervention



onebillion is an app-based mathematics learning programme for three-six year olds. A ‘virtual teacher’ provides oral explanations and visual demonstration of each topic, and then the child works through a number of activities for each topic before completing an end of topic quiz. Feedback is given after the child’s answer. Children work through the apps at their own pace and sessions are supervised by an adult (usually a TA). The EEF conducted a randomised controlled trial of the approach⁴⁶ in Year 1 with 113 schools, after some initial promising evidence including another RCT in England.⁴⁷ Children who had been identified by their teachers as being in the lower half of the class for mathematics attainment at the start of the school year took part. Children who received *onebillion* made an additional three months’ progress in mathematics compared to the control group. Furthermore, there was an indication that TAs played a crucial role in its successful deployment. *onebillion* is an EEF Promising Project.



GLOSSARY

Term	Meaning
<i>Conceptual understanding</i>	Understanding of abstract ideas in mathematics, such as numbers, operations, and relations.
<i>Developmental progression</i>	Description of a typical path that children tend to follow in developing an understanding of a mathematical topic.
<i>Executive functions</i>	The set of cognitive skills that control our thoughts and behaviour and have been found to be associated with learning. This includes working memory (maintaining and manipulating information in mind), inhibition (ignoring distractions and suppressing unwanted responses), and cognitive flexibility (switching between task or considering multiple perspectives of a situation).
<i>Self-regulation</i>	The ability to manage one's own behaviour and learning.
<i>Metacognition</i>	The ability to reflect on your own learning processes.
<i>Spatial thinking</i>	Thinking about relations between objects and space.
<i>Subitising</i>	<p>Perceptual subitising is the ability to instantly recognise the number of objects in a small group, without counting.</p> <p>Conceptual subitising is the ability to see a whole quantity as groups of smaller quantities (for example, seeing six as two groups of three).</p>
<i>Part-whole relations</i>	Refers to how numbers can be split into parts; for example, ten can be split into four and six.
<i>Cardinal principle</i>	Understanding that the last number spoken in a counting sequence names the quantity for that set. One of the five principles of counting established by Gelman and Galistel (1978) ⁴⁸
<i>Inverse relation</i>	An opposite relationship, for example addition and subtraction.
<i>Manipulative</i>	An object that children or practitioners can interact with and move to support the teaching and learning of mathematics.
<i>Representation</i>	Refers to a particular form in which mathematics is presented. These include informal drawings, mathematical symbols, and more formal diagrams such as a drawn number line or graph.
<i>Hierarchical inclusion</i>	The principle that each successive number is equal to the previous number plus one.

Term	Meaning
Number bond	Pairs of numbers that make up a given number that show how numbers are split or combined. For example number bonds to 5 are '1 and 4', and '2 and 3'.
Multiplicative composition	The principle that a given number can be expressed as factor pairs, for example, 8 can be expressed as '2 x 4' and '1 x 8'.
Guided interaction	Occurs when an adult and child collaborate on a task and the adult's strategies are highly tuned to the child's capabilities and motivations. ²⁷ The adult is responsive to the child's intentions, focuses on spontaneous learning, and provides opportunities for the child's feedback. Discussion is a key feature of this approach and the use of a variety of questions helps to develop and extend children's thinking.
Sustained shared thinking	<p>Involves two or more people working together to solve a problem, clarify an issue, evaluate activities, or extend a narrative.⁴⁹ Key features include all parties contributing to the interaction, with the aim of extending and developing children's thinking. Techniques that practitioners might use include:</p> <ul style="list-style-type: none"> • tuning in—listening carefully to what is being said and observing what the child is doing; • showing genuine interest—giving whole attention, eye contact, and smiling and nodding; • asking children to elaborate—'I really want to know more about this'; • recapping—'So you think that...'; • giving their own experience—'I find it useful to move things over as I count them to help me remember them'; • clarifying ideas—'So you think if I turn this round it will make a square?'; • using encouragement to extend thinking—'You have thought really hard about your tower, but what can you do next?'; • suggesting—'You might want to try doing it like this'; • reminding—'Don't forget that you said if you turn it round it will make a square'; and • asking open questions—'How did you?', 'Why does this...?', 'What happens next?'

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HOW WAS THIS GUIDANCE COMPILED

This guidance draws on the best available evidence regarding the teaching of mathematics from ages three to seven.

The report was developed over several stages.

The initial scoping stage began with a consultation with practitioners, academics, and other experts. The EEF team appointed an expert advisory panel and an evidence review team. A review of the evidence on mathematics teaching was conducted by Professor Jeremy Hodgen, Dr Nancy Barclay, Dr Colin Foster, Dr Camilla Gilmore, Dr Rachel Marks, and Dr Victoria Sims.

The evidence review is largely based on meta-analyses of randomised controlled trial and quasi-experimental design studies of interventions in order to investigate what actions can be taken by educators to improve mathematics outcomes for children aged three to seven. In addition, the models of learning and developmental progressions in Recommendation 1 are based on best evidence syntheses of a wider range of study designs. The guidance is also informed by a practice guide created by the U.S. What Works Clearinghouse 'Teaching Math to Young Children' and the EEF's Early Years Toolkit.

The EEF worked with the support of the advisory panel to draft the recommendations using their expert judgement. We would like to particularly thank Dr Sue Gifford for leading on the writing of examples in this guidance. Academics and teaching experts were consulted on drafts of the report.

NOTES

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Education Endowment Foundation
5th Floor, Millbank Tower
21-24 Millbank
London
SW1P 4QP

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